

Experimental Constraints on Four-Neutrino Mixing

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Abstract

It is shown that at least four massive neutrinos are needed in order to accommodate the evidences in favor of neutrino oscillations found in solar and atmospheric neutrino experiments and in the LSND experiment. Among all four-neutrino schemes, only two are compatible with the results of all neutrino oscillation experiments. These two schemes have a mass spectrum composed of two pairs of neutrinos with close masses separated by the “LSND gap” of the order of 1 eV.

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I. INTRODUCTION

Neutrino oscillations have been proposed by B. Pontecorvo [1] more than forty years ago from an analogy with $K^0 \rightleftharpoons \bar{K}^0$ oscillations. In 1967 B. Pontecorvo predicted the possibility that the flux of solar ν_e 's could be suppressed because of neutrino oscillations [2]. About one year later [3] R. Davis and collaborators reported the first measurement of the Homestake $^{37}\text{Cl}(\nu_e, e^-)^{37}\text{Ar}$ radiochemical detector, which gave an upper limit for the flux of solar ν_e 's on the Earth significantly smaller than the prediction of the existing Standard Solar Model [4]. This was the first experimental indication in favor of neutrino oscillations.

It is interesting to notice that in the 1967 paper [2] B. Pontecorvo introduced the concept of “sterile” neutrinos. He considered the possibility of oscillations of left-handed neutrinos created in weak interaction processes into left-handed “antineutrino” states, quanta of the right-handed component of the neutrino field that does not participate in weak interactions. He called these states “sterile”, opposed to the usual “active” neutrinos. He also pointed out that in this case the mass eigenstates are Majorana particles. As we will see later, the present experimental data indicate the existence of at least one sterile neutrino.

In 1969 V. Gribov and B. Pontecorvo wrote a famous paper in which they formulated the theory of $\nu_e \rightarrow \nu_\mu$ oscillations. In 1976 S.M. Bilenky and B. Pontecorvo [6] introduced the general scheme with Dirac and Majorana neutrino mass terms, laying the foundations of the theory of neutrino mixing. The early studies of neutrino mixing and neutrino oscillations are beautifully summarized in the 1978 review of S.M. Bilenky and B. Pontecorvo [7].

Today neutrino oscillations are subject to intensive experimental and theoretical research [8,9]. This beautiful quantum mechanical phenomenon provides information on the masses and mixing of neutrinos and is considered to be one of the best ways to explore the physics beyond the Standard Model. Indeed, the smallness of neutrino masses may be due to the existence of a very large energy scale at which the conservation of lepton number is violated and small Majorana neutrino masses are generated through the see-saw mechanism [10] or through non-renormalizable interaction terms in the effective Lagrangian of the Standard Model [11].

The best evidence in favor of the existence of neutrino oscillations has been recently provided by the measurement in the Super-Kamiokande experiment [12] of an up-down asymmetry of high-energy μ -like events generated by atmospheric neutrinos:

$$\mathcal{A}_\mu \equiv (D_\mu - U_\mu)/(D_\mu + U_\mu) = 0.311 \pm 0.043 \pm 0.01. \quad (1.1)$$

Here D_μ and U_μ are, respectively, the number of downward-going and upward-going events, corresponding to the zenith angle intervals $0.2 < \cos \theta < 1$ and $-1 < \cos \theta < -0.2$. Since the fluxes of high-energy downward-going and upward-going atmospheric neutrinos are predicted to be equal with high accuracy on the basis of geometrical arguments [13], the Super-Kamiokande evidence in favor of neutrino oscillations is model-independent. It provides a confirmation of the indications in favor of oscillations of atmospheric neutrinos found through the measurement of the ratio of μ -like and e -like events (Kamiokande, IMB, Soudan 2, Super-Kamiokande) [14,12] and through the measurement of upward-going muons produced by neutrino interactions in the rock below the detector (Super-Kamiokande, MACRO) [15]. Large $\nu_\mu \rightleftharpoons \nu_e$ oscillations of atmospheric neutrinos are excluded by the absence of a

up–down asymmetry of high-energy e -like events generated by atmospheric neutrinos and detected in the Super-Kamiokande experiment ($\mathcal{A}_e = 0.036 \pm 0.067 \pm 0.02$) [12] and by the negative result of the CHOOZ long-baseline $\bar{\nu}_e$ disappearance experiment [16]. Therefore, the atmospheric neutrino anomaly consists in the disappearance of muon neutrinos and can be explained by $\nu_\mu \rightarrow \nu_\tau$ and/or $\nu_\mu \rightarrow \nu_s$ oscillations (here ν_s is a sterile neutrino that does not take part in weak interactions).

Other indications in favor of neutrino oscillations have been obtained in solar neutrino experiments (Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande) [17] and in the LSND experiment [18].

The flux of electron neutrinos measured in all five solar neutrino experiments is substantially smaller than the one predicted by the Standard Solar Model [19] and a comparison of the data of different experiments indicate an energy dependence of the solar ν_e suppression, which represents a rather convincing evidence in favor of neutrino oscillations [9]. The disappearance of solar electron neutrinos can be explained by $\nu_e \rightarrow \nu_\mu$ and/or $\nu_e \rightarrow \nu_\tau$ and/or $\nu_e \rightarrow \nu_s$ oscillations [20].

The accelerator LSND experiment is the only one that claims the observation of neutrino oscillations in specific appearance channels: $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$. Since the appearance of neutrinos with a different flavor represents the true essence of neutrino oscillations, the LSND evidence is extremely interesting and its confirmation (or disproof) by other experiments should receive high priority in future research. Four such experiments have been proposed and are under study: BooNE at Fermilab, I-216 at CERN, ORLaND at Oak Ridge and NESS at the European Spallation Source [21]. Among these proposals only BooNE is approved and will start in 2001.

Neutrino oscillations occur if neutrinos are massive and mixed particles [7–9], *i.e.* if the left-handed components $\nu_{\alpha L}$ of the flavor neutrino fields are superpositions of the left-handed components ν_{kL} ($k = 1, \dots, N$) of neutrino fields with definite mass m_k :

$$\nu_{\alpha L} = \sum_{k=1}^N U_{\alpha k} \nu_{kL}, \quad (1.2)$$

where U is a $N \times N$ unitary mixing matrix. From the measurement of the invisible decay width of the Z -boson it is known that the number of light active neutrino flavors is three [22], corresponding to ν_e , ν_μ and ν_τ . This implies that the number N of massive neutrinos is bigger or equal to three. If $N > 3$, in the flavor basis there are $N_s = N - 3$ sterile neutrinos, $\nu_{s_1}, \dots, \nu_{s_{N_s}}$. In this case the flavor index α in Eq. (1.2) takes the values $e, \mu, \tau, s_1, \dots, s_{N_s}$.

II. THE NECESSITY OF AT LEAST THREE INDEPENDENT Δm^2 'S

The three evidences in favor of neutrino oscillations found in solar and atmospheric neutrino experiments and in the accelerator LSND experiment imply the existence of at least three independent neutrino mass-squared differences. This can be seen by considering the general expression for the probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions in vacuum, that can be written as [7–9]

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \sum_{k=1}^N U_{\alpha k}^* U_{\beta k} \exp \left(-i \frac{\Delta m_{kj}^2 L}{2 E} \right) \right|^2, \quad (2.1)$$

where $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$, j is any of the mass-eigenstate indices, L is the distance between the neutrino source and detector and E is the neutrino energy. The range of L/E characteristic of each type of experiment is different: $L/E \sim 10^{11} - 10^{12} \text{ eV}^{-2}$ for solar neutrino experiments, $L/E \sim 10^2 - 10^3 \text{ eV}^{-2}$ for atmospheric neutrino experiments and $L/E \sim 1 \text{ eV}^{-2}$ for the LSND experiment. From Eq. (2.1) it is clear that neutrino oscillations are observable in an experiment only if there is at least one mass-squared difference Δm_{kj}^2 such that

$$\frac{\Delta m_{kj}^2 L}{2 E} \gtrsim 0.1 \quad (2.2)$$

(the precise lower bound depends on the sensitivity of the experiment) in a significant part of the energy and source-detector distance intervals of the experiment (if the condition (2.2) is not satisfied, $P_{\nu_\alpha \rightarrow \nu_\beta} \simeq |\sum_k U_{\alpha k}^* U_{\beta k}|^2 = \delta_{\alpha\beta}$). Since the range of L/E probed by the LSND experiment is the smaller one, a large mass-squared difference is needed for LSND oscillations:

$$\Delta m_{\text{LSND}}^2 \gtrsim 10^{-1} \text{ eV}^2. \quad (2.3)$$

Specifically, the maximum likelihood analysis of the LSND data in terms of two-neutrino oscillations gives [18]

$$0.20 \text{ eV}^2 \lesssim \Delta m_{\text{LSND}}^2 \lesssim 2.0 \text{ eV}^2. \quad (2.4)$$

Furthermore, from Eq. (2.1) it is clear that a dependence of the oscillation probability from the neutrino energy E and the source-detector distance L is observable only if there is at least one mass-squared difference Δm_{kj}^2 such that

$$\frac{\Delta m_{kj}^2 L}{2 E} \sim 1. \quad (2.5)$$

Indeed, all the phases $\Delta m_{kj}^2 L/2E \gg 1$ are washed out by the average over the energy and source-detector ranges characteristic of the experiment. Since a variation of the oscillation probability as a function of neutrino energy has been observed both in solar and atmospheric neutrino experiments and the ranges of L/E characteristic of these two types of experiments are different from each other and different from the LSND range, two more mass-squared differences with different scales are needed:

$$\Delta m_{\text{sun}}^2 \sim 10^{-12} - 10^{-11} \text{ eV}^2 \quad (\text{VO}), \quad (2.6)$$

$$\Delta m_{\text{atm}}^2 \sim 10^{-3} - 10^{-2} \text{ eV}^2. \quad (2.7)$$

The condition (2.6) for the solar mass-squared difference Δm_{sun}^2 has been obtained under the assumption of vacuum oscillations (VO). If the disappearance of solar ν_e 's is due to the MSW effect [23], the condition

$$\Delta m_{\text{sun}}^2 \lesssim 10^{-4} \text{ eV}^2 \quad (\text{MSW}) \quad (2.8)$$

must be fulfilled in order to have a resonance in the interior of the sun. Hence, in the MSW case Δm_{sun}^2 must be at least one order of magnitude smaller than Δm_{atm}^2 .

It is possible to ask if three different scales of neutrino mass-squared differences are needed even if the results of the Homestake solar neutrino experiment is neglected, allowing an energy-independent suppression of the solar ν_e flux. The answer is that still the data cannot be fitted with only two neutrino mass-squared differences because an energy-independent suppression of the solar ν_e flux requires large $\nu_e \rightarrow \nu_\mu$ or $\nu_e \rightarrow \nu_\tau$ transitions generated by Δm_{atm}^2 or Δm_{LSND}^2 . These transitions are forbidden by the results of the Bugey [24] and CHOOZ [16] reactor $\bar{\nu}_e$ disappearance experiments and by the non-observation of an up-down asymmetry of e -like events in the Super-Kamiokande atmospheric neutrino experiment [12].

III. FOUR-NEUTRINO SCHEMES

The existence of three different scales of Δm^2 imply that at least four light massive neutrinos must exist in nature. Here we consider the schemes with four light and mixed neutrinos [25–40], which constitute the minimal possibility that allows to explain all the existing data with neutrino oscillations. In this case, in the flavor basis the three active neutrinos ν_e, ν_μ, ν_τ are accompanied by a sterile neutrino ν_s . Let us notice that the existence of four light massive neutrinos is favored also by the possibility that active-sterile neutrino transitions in neutrino-heated supernova ejecta could enable the production of r -process nuclei [41].

The six types of four-neutrino mass spectra with three different scales of Δm^2 that can accommodate the hierarchy $\Delta m_{\text{sun}}^2 \ll \Delta m_{\text{atm}}^2 \ll \Delta m_{\text{LSND}}^2$ are shown qualitatively in Fig. 1. In all these mass spectra there are two groups of close masses separated by the “LSND gap” of the order of 1 eV. In each scheme the smallest mass-squared difference corresponds to Δm_{sun}^2 (Δm_{21}^2 in schemes I and B, Δm_{32}^2 in schemes II and IV, Δm_{43}^2 in schemes III and A), the intermediate one to Δm_{atm}^2 (Δm_{31}^2 in schemes I and II, Δm_{42}^2 in schemes III and IV, Δm_{21}^2 in scheme A, Δm_{43}^2 in scheme B) and the largest mass squared difference $\Delta m_{41}^2 = \Delta m_{\text{LSND}}^2$ is relevant for the oscillations observed in the LSND experiment. The six schemes are divided into four schemes of class 1 (I–IV) in which there is a group of three masses separated from an isolated mass by the LSND gap, and two schemes of class 2 (A, B) in which there are two couples of close masses separated by the LSND gap.

IV. THE DISFAVORED SCHEMES OF CLASS 1

In the following we will show that the schemes of class 1 are disfavored by the data if also the negative results of short-baseline accelerator and reactor disappearance neutrino oscillation experiments are taken into account [28,34,35]. Let us remark that in principle one could check which schemes are allowed by doing a combined fit of all data in the framework of the most general four-neutrino mixing scheme, with three mass-squared differences, six mixing angles and three CP-violating phases as free parameters. However, at the moment it is not possible to perform such a fit because of the enormous complications due to the

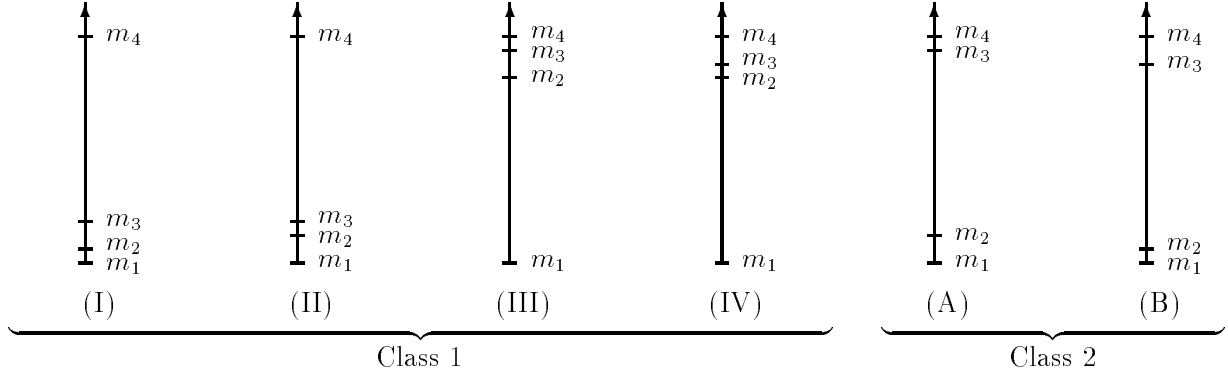


Figure 1

presence of too many parameters and to the difficulties involved in a combined fit of the data of different experiments, which are usually analyzed by the experimental collaborations using different methods. Hence, we think that it is quite remarkable that one can exclude the schemes of class 1 with the following relatively simple procedure.

Let us define the quantities d_α , with $\alpha = e, \mu, \tau, s$, in the schemes of class 1 as

$$d_\alpha^{(I)} \equiv |U_{\alpha 4}|^2, \quad d_\alpha^{(II)} \equiv |U_{\alpha 4}|^2, \quad d_\alpha^{(III)} \equiv |U_{\alpha 1}|^2, \quad d_\alpha^{(IV)} \equiv |U_{\alpha 1}|^2. \quad (4.1)$$

Physically d_α quantifies the mixing of the flavor neutrino ν_α with the isolated neutrino, whose mass is separated from the other three by the LSND gap.

The probability of $\nu_\alpha \rightarrow \nu_\beta$ ($\beta \neq \alpha$) and $\nu_\alpha \rightarrow \nu_\alpha$ transitions (and the corresponding probabilities for antineutrinos) in short-baseline experiments are given by [28,9]

$$P_{\nu_\alpha \rightarrow \nu_\beta} = A_{\alpha;\beta} \sin^2 \frac{\Delta m_{41}^2 L}{4E}, \quad P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - B_{\alpha;\alpha} \sin^2 \frac{\Delta m_{41}^2 L}{4E}, \quad (4.2)$$

with the oscillation amplitudes

$$A_{\alpha;\beta} = 4 d_\alpha d_\beta, \quad B_{\alpha;\alpha} = 4 d_\alpha (1 - d_\alpha). \quad (4.3)$$

The probabilities (4.2) have the same form as the corresponding probabilities in the case of two-neutrino mixing, $P_{\nu_\alpha \rightarrow \nu_\beta} = \sin^2(2\vartheta) \sin^2(\Delta m^2 L/4E)$ and $P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \sin^2(2\vartheta) \sin^2(\Delta m^2 L/4E)$, which have been used by all experimental collaborations for the analysis of the data in order to get information on the parameters $\sin^2(2\vartheta)$ and Δm^2 (ϑ and Δm^2 are, respectively, the mixing angle and the mass-squared difference in the case of two-neutrino mixing). Therefore, we can use the results of their analyses in order to get information on the corresponding parameters $A_{\alpha;\beta}$, $B_{\alpha;\alpha}$ and Δm_{41}^2 .

The exclusion plots obtained in short-baseline $\bar{\nu}_e$ and ν_μ disappearance experiments imply that [28]

$$d_\alpha \leq a_\alpha^0 \quad \text{or} \quad d_\alpha \geq 1 - a_\alpha^0 \quad (\alpha = e, \mu), \quad (4.4)$$

with

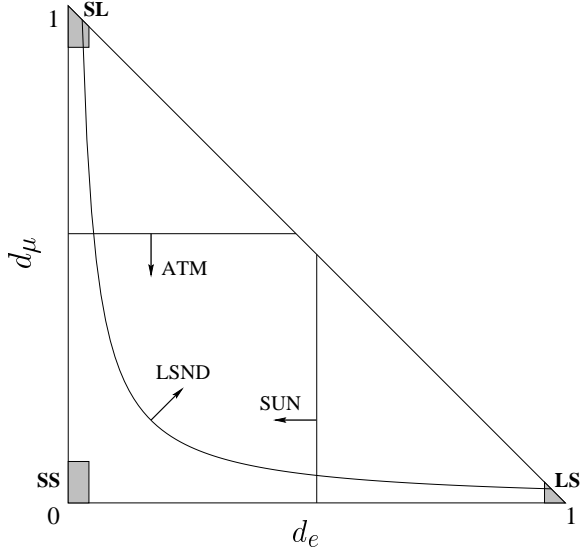


Figure 2

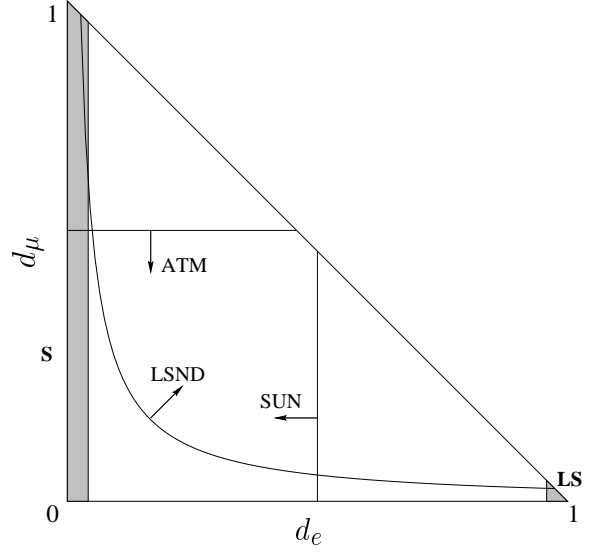


Figure 3

$$a_\alpha^0 = \frac{1}{2} \left(1 - \sqrt{1 - B_{\alpha;\alpha}^0} \right) \quad (\alpha = e, \mu), \quad (4.5)$$

where $B_{e;e}^0$ and $B_{\mu;\mu}^0$ are the upper bounds, that depend on Δm_{41}^2 , of the oscillation amplitudes $B_{e;e}$ and $B_{\mu;\mu}$ given by the exclusion plots of $\bar{\nu}_e$ and ν_μ disappearance experiments. From the exclusion curves of the Bugey reactor $\bar{\nu}_e$ disappearance experiment [24] and of the CDHS and CCFR accelerator ν_μ disappearance experiments [42] it follows that $a_e^0 \lesssim 3 \times 10^{-2}$ for $\Delta m_{41}^2 = \Delta m_{\text{LSND}}^2$ in the LSND range (2.4) and $a_\mu^0 \lesssim 0.2$ for $\Delta m_{41}^2 \gtrsim 0.4 \text{ eV}^2$ [9].

Therefore, the negative results of short-baseline $\bar{\nu}_e$ and ν_μ disappearance experiments imply that d_e and d_μ are either small or large (close to one). Taking into account the unitarity limit $d_e + d_\mu \leq 1$, for each value of Δm_{41}^2 above about 0.3 eV^2 there are three regions in the d_e - d_μ plane that are allowed by the results of disappearance experiments: region SS with small d_e and d_μ , region LS with large d_e and small d_μ and region SL with small d_e and large d_μ . These three regions are illustrated qualitatively by the three shadowed areas in Fig. 2. For $\Delta m_{41}^2 \lesssim 0.3 \text{ eV}^2$ there is no constraint on the value of d_μ from the results of short-baseline ν_μ disappearance experiments and there are two regions in the d_e - d_μ plane that are allowed by the results of $\bar{\nu}_e$ disappearance experiments: region S with small d_e and region LS with large d_e and small d_μ (the smallness of d_μ follows from the unitarity bound $d_e + d_\mu \leq 1$). These two regions are illustrated qualitatively by the two shadowed areas in Fig. 3.

Let us consider now the results of solar neutrino experiments, which imply a disappearance of electron neutrinos. The survival probability of solar ν_e 's averaged over the fast unobservable oscillations due to Δm_{41}^2 and Δm_{31}^2 is bounded by [28,9]

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \geq d_e^2. \quad (4.6)$$

Therefore, only the possibility

$$d_e \leq a_e^0 \quad (4.7)$$

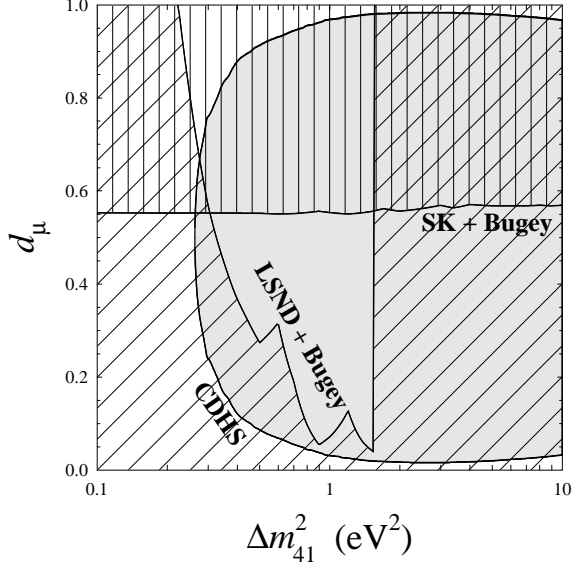


Figure 4

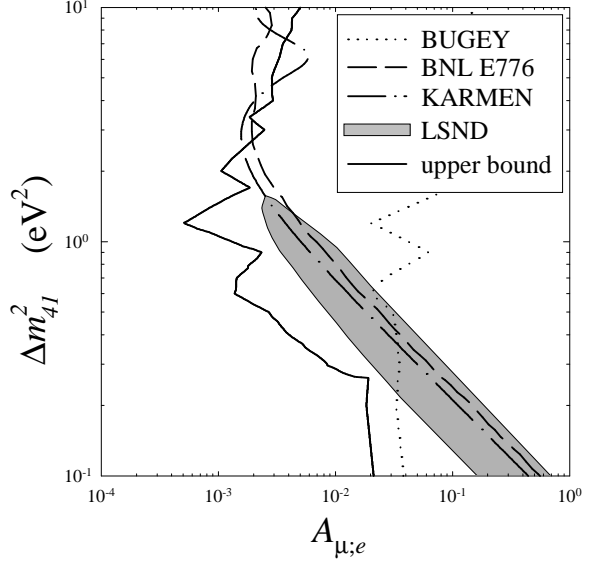


Figure 5

is acceptable in order to explain the observed deficit of solar ν_e 's with neutrino oscillations. Indeed, the solar neutrino data imply an upper bound for d_e , that is shown qualitatively by the vertical lines in Figs. 2 and 3. It is clear that the regions LS in Figs. 2 and 3 are disfavored by the results of solar neutrino experiments.

In a similar way, since the survival probability of atmospheric ν_μ 's and $\bar{\nu}_\mu$'s is bounded by [28,9]

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{atm}} \geq d_\mu^2, \quad (4.8)$$

large values of d_μ are incompatible with the asymmetry (1.1) observed in the Super-Kamiokande experiment. The upper bound for d_μ that follows from atmospheric neutrino data is shown qualitatively by the horizontal lines in Figs. 2 and 3. It is clear that the region SL in Fig. 2, that is allowed by the results of ν_μ short-baseline disappearance experiments for $\Delta m_{41}^2 \gtrsim 0.3 \text{ eV}^2$, and the large- d_μ part of the region S in Fig. 3 are disfavored by the results of atmospheric neutrino experiments.

Therefore, only the region SS in Fig. 2 and the small- d_μ part of the region S in Fig. 3 are allowed by the results of solar and atmospheric neutrino experiments. In both cases d_μ is small. But such small values of d_μ are disfavored by the results of the LSND experiment, that imply a lower bound $A_{\mu;e}^{\text{min}}$ for the amplitude $A_{\mu;e} = 4d_e d_\mu$ of $\nu_\mu \rightarrow \nu_e$ oscillations. Indeed, we have

$$d_e d_\mu \geq A_{\mu;e}^{\text{min}}/4. \quad (4.9)$$

This bound, shown qualitatively by the LSND exclusion curves in Figs. 2 and 3, excludes region SS in Fig. 2 and the small- d_μ part of region S in Fig. 3. From Figs. 2 and 3 one can see in a qualitative way that in the schemes of class 1 the results of the solar, atmospheric and LSND experiments are incompatible with the negative results of short-baseline experiments.

A quantitative illustration of this incompatibility is given in Fig. 4, in which the shadowed area is excluded by the bound $d_\mu \leq a_\mu^0$ or $d_\mu \geq 1 - a_\mu^0$ obtained from the exclusion plot of the

short-baseline CDHS ν_μ disappearance experiment. The horizontal line in Fig. 4 represents the upper bound

$$d_\mu \lesssim 0.55 \equiv a_\mu^{\text{SK}} \quad (4.10)$$

(the vertically hatched area above the line is excluded) obtained from the Super-Kamiokande asymmetry (1.1) and the exclusion curve of the Bugey $\bar{\nu}_e$ disappearance experiment [35]. It is clear that the results of short-baseline disappearance experiments and the Super-Kamiokande asymmetry (1.1) imply that $d_\mu \lesssim 0.55$ for $\Delta m_{41}^2 \lesssim 0.3 \text{ eV}^2$ and that d_μ is very small for $\Delta m_{41}^2 \gtrsim 0.3 \text{ eV}^2$. These small values of d_μ are disfavored by the curve in Fig. 4 labelled LSND + Bugey, which represents the constraint

$$d_\mu \geq A_{\mu;e}^{\text{min}} / 4a_e^0 \quad (4.11)$$

(the diagonally hatched area is excluded), derived from the inequality (4.9) using the bound (4.7). Hence, in the framework of the schemes of class 1 there is no range of d_μ that is compatible with all the experimental data.

Another way for seeing the incompatibility of the experimental results with the schemes of class 1 is presented in Fig. 5, where we have plotted in the $A_{\mu;e}-\Delta m_{41}^2$ plane the upper bound $A_{\mu;e} \leq 4a_e^0 a_\mu^0$ for $\Delta m_{41}^2 > 0.26 \text{ eV}^2$ and $A_{\mu;e} \leq 4a_e^0 a_\mu^{\text{SK}}$ for $\Delta m_{41}^2 < 0.26 \text{ eV}^2$ (solid line, the region on the right is excluded). One can see that this constraint is incompatible with the LSND-allowed region (shadowed area).

Summarizing, we have reached the conclusion that the four schemes of class 1 shown in Fig. 1 are disfavored by the data.

V. THE FAVORED SCHEMES OF CLASS 2

The four-neutrino schemes of class 2 are compatible with the results of all neutrino oscillation experiments if the mixing of ν_e with the two mass eigenstates responsible for the oscillations of solar neutrinos (ν_3 and ν_4 in scheme A and ν_1 and ν_2 in scheme B) is large and the mixing of ν_μ with the two mass eigenstates responsible for the oscillations of atmospheric neutrinos (ν_1 and ν_2 in scheme A and ν_3 and ν_4 in scheme B) is large [27,28,34,35]. This is illustrated qualitatively in Figs. 6 and 7, as we are going to explain.

Let us define the quantities c_α , with $\alpha = e, \mu, \tau, s$, in the schemes A and B as

$$c_\alpha^{(\text{A})} \equiv \sum_{k=1,2} |U_{\alpha k}|^2, \quad c_\alpha^{(\text{B})} \equiv \sum_{k=3,4} |U_{\alpha k}|^2. \quad (5.1)$$

Physically c_α quantify the mixing of the flavor neutrino ν_α with the two massive neutrinos whose Δm^2 is relevant for the oscillations of atmospheric neutrinos (ν_1, ν_2 in scheme A and ν_3, ν_4 in scheme B). The exclusion plots obtained in short-baseline $\bar{\nu}_e$ and ν_μ disappearance experiments imply that [28]

$$c_\alpha \leq a_\alpha^0 \quad \text{or} \quad c_\alpha \geq 1 - a_\alpha^0 \quad (\alpha = e, \mu), \quad (5.2)$$

with a_α^0 given in Eq. (4.5).

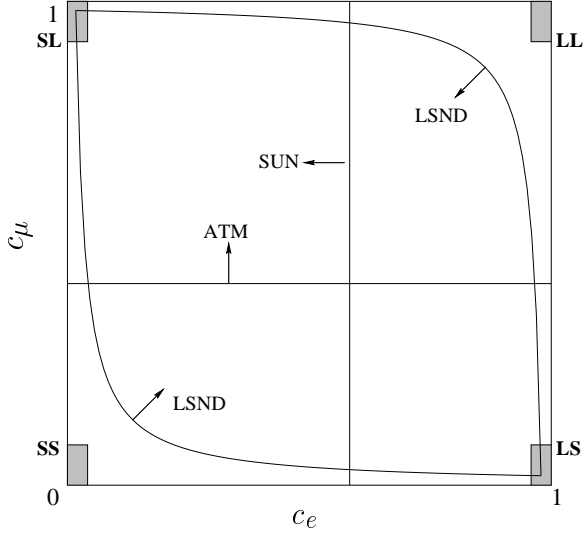


Figure 6

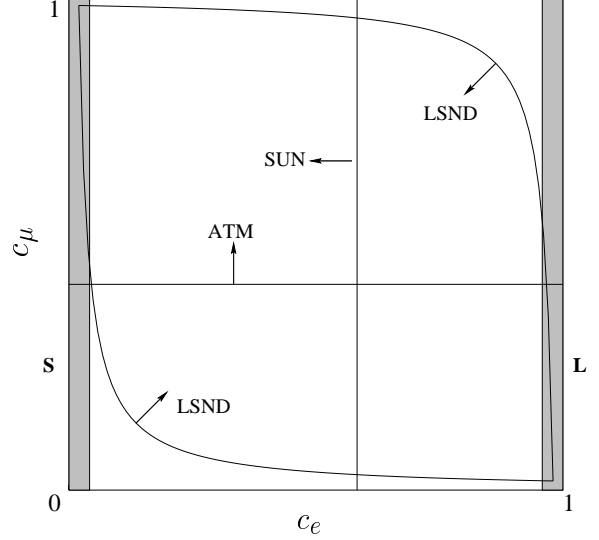


Figure 7

The shadowed areas in Figs. 6 and 7 illustrate qualitatively the regions in the c_e - c_μ plane allowed by the negative results of short-baseline $\bar{\nu}_e$ and ν_μ disappearance experiments. Figure 6 is valid for $\Delta m_{41}^2 \gtrsim 0.3 \text{ eV}^2$ and shows that there are four regions allowed by the results of short-baseline disappearance experiments: region SS with small c_e and c_μ , region LS with large c_e and small c_μ , region SL with small c_e and large c_μ and region LL with large c_e and c_μ . The quantities c_e and c_μ can be both large, because the unitarity of the mixing matrix imply that $c_\alpha + c_\beta \leq 2$ and $0 \leq c_\alpha \leq 1$ for $\alpha, \beta = e, \mu, \tau, s$. Figure 7 is valid for $\Delta m_{41}^2 \lesssim 0.3 \text{ eV}^2$, where there is no constraint on the value of c_μ from the results of short-baseline ν_μ disappearance experiments. It shows that there are two regions allowed by the results of short-baseline $\bar{\nu}_e$ disappearance experiments: region S with small c_e and region L with large c_e .

Let us take now into account the results of solar neutrino experiments. Large values of c_e are incompatible with solar neutrino oscillations because in this case ν_e has large mixing with the two massive neutrinos responsible for atmospheric neutrino oscillations and, through the unitarity of the mixing matrix, small mixing with the two massive neutrinos responsible for solar neutrino oscillations. Indeed, in the schemes of class 2 the survival probability $P_{\nu_e \rightarrow \nu_e}^{\text{sun}}$ of solar ν_e 's is bounded by [28,9]

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \geq c_e^2/2, \quad (5.3)$$

and its possible variation $\Delta P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E)$ with neutrino energy E is limited by [28,9]

$$\Delta P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \leq (1 - c_e)^2. \quad (5.4)$$

If c_e is large as in the LS or LL regions of Fig. 6 or in the L region of Fig. 7, we have

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \geq \frac{(1 - a_e^0)^2}{2} \simeq \frac{1}{2}, \quad \Delta P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \leq (a_e^0)^2 \lesssim 10^{-3}, \quad (5.5)$$

for $\Delta m_{41}^2 = \Delta m_{\text{LSND}}^2$ in the LSND range (2.4). Therefore $P_{\nu_e \rightarrow \nu_e}^{\text{sun}}$ is bigger than 1/2 and practically does not depend on neutrino energy. Since this is incompatible with the results

of solar neutrino experiments interpreted in terms of neutrino oscillations [9,20], we conclude that the regions LS and LL in Fig. 6 and the region L in Fig. 7 are disfavored by solar neutrino data, as illustrated qualitatively by the vertical exclusion lines in Figs. 6 and 7.

Let us consider now the results of atmospheric neutrino experiments. Small values of c_μ are incompatible with atmospheric neutrino oscillations because in this case ν_μ has small mixing with the two massive neutrinos responsible for atmospheric neutrino oscillations. Indeed, the survival probability of atmospheric ν_μ 's is bounded by [28,9]

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{atm}} \geq (1 - c_\mu)^2, \quad (5.6)$$

and it can be shown [35] that the Super-Kamiokande asymmetry (1.1) and the exclusion curve of the Bugey $\bar{\nu}_e$ disappearance experiment imply the upper bound

$$c_\mu \gtrsim 0.45 \equiv b_\mu^{\text{SK}}. \quad (5.7)$$

This limit is depicted qualitatively by the horizontal exclusion lines in Figs. 6 and 7. Therefore, we conclude that the regions SS and LS in Fig. 6 and the small- c_μ parts of the regions S and L in Fig. 7 are disfavored by atmospheric neutrino data.

Finally, let us consider the results of the LSND experiment. In the schemes of class 2 the amplitude of short-baseline $\nu_\mu \rightarrow \nu_e$ oscillations is given by

$$A_{\mu;e} = \left| \sum_{k=1,2} U_{ek} U_{\mu k}^* \right|^2 = \left| \sum_{k=3,4} U_{ek} U_{\mu k}^* \right|^2. \quad (5.8)$$

The second equality in Eq. (5.8) is due to the unitarity of the mixing matrix. Using the Cauchy-Schwarz inequality we obtain

$$c_e c_\mu \geq A_{\mu;e}^{\text{min}}/4 \quad \text{and} \quad (1 - c_e)(1 - c_\mu) \geq A_{\mu;e}^{\text{min}}/4, \quad (5.9)$$

where $A_{\mu;e}^{\text{min}}$ is the minimum value of the oscillation amplitude $A_{\mu;e}$ observed in the LSND experiment. The bounds (5.9) are illustrated qualitatively in Figs. 6 and 7. One can see that the results of the LSND experiment confirm the exclusion of the regions SS and LL in Fig. 6 and the exclusion of the small- c_μ part of region S and of the large- c_μ part of region L in Fig. 7.

Summarizing, if $\Delta m_{41}^2 \gtrsim 0.3 \text{ eV}^2$ only the region SL in Fig. 6, with

$$c_e \leq a_e^0 \quad \text{and} \quad c_\mu \geq 1 - a_\mu^0, \quad (5.10)$$

is compatible with the results of all neutrino oscillation experiments. If $\Delta m_{41}^2 \lesssim 0.3 \text{ eV}^2$ only the large- c_μ part of region S in Fig. 7, with

$$c_e \leq a_e^0 \quad \text{and} \quad c_\mu \geq b_\mu^{\text{SK}}, \quad (5.11)$$

is compatible with the results of all neutrino oscillation experiments. Therefore, in any case c_e is small and c_μ is large. However, it is important to notice that, as shown clearly in Figs. 6 and 7, the inequalities (5.9) following from the LSND observation of short-baseline $\nu_\mu \rightarrow \nu_e$ oscillations imply that c_e , albeit small, has a lower bound and c_μ , albeit large, has an upper bound:

$$c_e \gtrsim A_{\mu;e}^{\text{min}}/4 \quad \text{and} \quad c_\mu \lesssim 1 - A_{\mu;e}^{\text{min}}/4. \quad (5.12)$$

VI. CONCLUSIONS

We have seen that only the two four-neutrino schemes A and B of class 2 in Fig. 1 are compatible with the results of all neutrino oscillation experiments. Furthermore, we have shown that the quantities c_e and c_μ in these two schemes must be, respectively, small and large. Physically c_α , defined in Eq. (5.1), quantify the mixing of the flavor neutrino ν_α with the two massive neutrinos whose Δm^2 is relevant for the oscillations of atmospheric neutrinos (ν_1, ν_2 in scheme A and ν_3, ν_4 in scheme B).

The smallness of c_e implies that electron neutrinos do not oscillate in atmospheric and long-baseline neutrino oscillation experiments. Indeed, one can obtain rather stringent upper bounds for the probability of ν_e transitions into any other state [29] and for the size of CP or T violation that could be measured in long-baseline experiments in the $\nu_\mu \rightleftharpoons \nu_e$ and $\bar{\nu}_\mu \rightleftharpoons \bar{\nu}_e$ channels [30].

Let us consider [36–39] now the effective Majorana mass in neutrinoless double- β decay,

$$|\langle m \rangle| = \left| \sum_{k=1}^4 U_{ek}^2 m_k \right|. \quad (6.1)$$

In scheme A, since c_e is small, the effective Majorana mass is approximately given by

$$|\langle m \rangle| \simeq |U_{e3}^2 + U_{e4}^2| m_4 \simeq |U_{e3}^2 + U_{e4}^2| \sqrt{\Delta m_{\text{LSND}}^2}. \quad (6.2)$$

Therefore, in scheme A the effective Majorana mass can be as large as $\sqrt{\Delta m_{\text{LSND}}^2}$. Since c_e is small, in scheme A we have $|U_{e3}|^2 \simeq \cos^2 \vartheta_{\text{sun}}$ and $|U_{e4}|^2 \simeq \sin^2 \vartheta_{\text{sun}}$, where ϑ_{sun} is the mixing angle determined from the two-generation analysis of solar neutrino data [20]. In the case of the small mixing angle MSW solution of the solar neutrino problem $|U_{e4}| \ll |U_{e3}| \simeq 1$ and from Eq. (6.2) one can see that $|\langle m \rangle| \simeq \sqrt{\Delta m_{\text{LSND}}^2}$. In the case of the large mixing angle MSW solution, $\sin^2 2\vartheta_{\text{sun}}$ is constrained to be less than about 0.97 at 99% CL [43] and the effective Majorana mass lies in the range [39] $7 \times 10^{-2} \text{ eV} \lesssim |\langle m \rangle| \lesssim 1.4 \text{ eV}$. In the case of the vacuum oscillation solution, $\sin^2 2\vartheta_{\text{sun}}$ can be as large as one and there is no lower bound for $|\langle m \rangle|$. If future measurements will show the correctness of a large mixing angle solution of the solar neutrino problem (due to vacuum oscillations or the MSW effect), the measurement of $|\langle m \rangle|$ would give information [27,39] on the value of a Majorana phase in the mixing matrix U , that does not contribute to neutrino oscillations [44].

In scheme B the contribution of the “heavy” neutrino masses m_3 and m_4 to the effective Majorana mass is strongly suppressed [36–39]:

$$|\langle m \rangle|_{34} \equiv |U_{e3}^2 m_3 + U_{e4}^2 m_4| \lesssim c_e m_4 \leq a_e^0 \sqrt{\Delta m_{\text{LSND}}^2} \simeq 2 \times 10^{-2} \text{ eV}. \quad (6.3)$$

Therefore, if future neutrinoless double- β decay experiments will find that $|\langle m \rangle| \gtrsim 2 \times 10^{-2} \text{ eV}$, it would mean that scheme B is excluded, or that neutrinoless double- β decay proceeds through other mechanisms, not involving the effective Majorana mass $|\langle m \rangle|$.

Finally, if the upper bound $N_\nu^{\text{BBN}} < 4$ for the effective number of neutrinos in Big-Bang Nucleosynthesis is correct [45], the mixing of ν_s with the two mass eigenstates responsible for the oscillations of atmospheric neutrinos must be very small [31–33]. In this case atmospheric

neutrinos oscillate only in the $\nu_\mu \rightarrow \nu_\tau$ channel and solar neutrinos oscillate only in the $\nu_e \rightarrow \nu_s$ channel. This is very important because it implies that the two-generation analyses of solar and atmospheric neutrino data give correct information on neutrino mixing in the two four-neutrino schemes A and B. Otherwise, it will be necessary to reanalyze the solar and atmospheric neutrino data using a general formalism that takes into account the possibility of simultaneous transitions into active and sterile neutrinos in solar and atmospheric neutrino experiments [40].

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